Foundations of Query Languages

Dr. Fang Wei

Lehrstuhl für Datenbanken und Informationssysteme
Universität Freiburg

SS 2011

Relational Model

- We assume that a countably infinite set **attr** of *attributes* is fixed.
- The *domain* is a countably infinite set **dom** (disjoint from **attr**).
- A *constant* is an element of **dom**.
- A relation schema is simply a relation name R, with $arity(R) = n$ (written as $R[n]$).
- A database schema is a nonempty finite set $\mathcal{R}$ of relation names.
- A *tuple* over a (possibly empty) finite set U of attributes (or over a relation schema $R[U]$) is a total mapping u from U to **dom**.
- or, a *tuple* is an ordered n-tuple ($n \geq 0$) of constants – an element of the Cartesian product $\mathbf{dom}^n$.
- A *database instance* is a finite set $\mathcal{I}$ that is the union of tuples.

Conjunctive Queries

Definition

A *conjunctive Query* Q over a database schema $\mathcal{R}$ is given as

$$ans(\vec{U}) \leftarrow R_1(\vec{U_1}), \ldots, R_n(\vec{U_n}),$$

such that for $1 \leq i \leq n$

- R_i a relation name in $\mathcal{R}$ and
- $\vec{U}$ and $\vec{U_i}$ vectors of variables and constants;
- any variable appearing in $\vec{U}$ appears also in some $\vec{U_i}$.
- Left to $\leftarrow$ is the *head* of the query, and to the right there is the *body*. The atoms in the body are also called *subgoals*.

Example

$$Sales(Part, Supplier, Customer),$$
$$Part(PName, Type),$$
$$Cust(CName, CAddr),$$
$$Supp(SName, SAddr).$$

$$Q: \quad ans(T) \leftarrow Sales(P, S, C), Part(P, T), Cust(C, A), Supp(S, A)$$

$$ans(\vec{U}) \leftarrow R_1(\vec{U_1}), \ldots, R_n(\vec{U_n}).$$

Answer

- The set of answers Q w.r.t. an instance $\mathcal{I}$ is denoted $Q(\mathcal{I})$.
- If there is a substitution (mapping) σ from the variables in $\vec{U_1}, \ldots, \vec{U_n}$ to the constants in **dom**, such that $\sigma(R_1(\vec{U_1})), \ldots, \sigma(R_n(\vec{U_n})) \in \mathcal{I}$, then by applying the same substitution σ to $\vec{U}$, we say that $\sigma(ans(\vec{U}))$ is an answer in $Q(\mathcal{I})$.
- Note that a substitution is a function such that a variable is mapped into only one constant, and a constant is mapped into itself.

$$ans(\vec{U}) \leftarrow R_1(\vec{U_1}), \ldots, R_n(\vec{U_n}).$$

Complexity of Query Answering

Let Q be a conjunctive query and $\mathcal{I}$ a database instance. what is the complexity of computing all the answers of $Q(\mathcal{I})$?

$$N^m$$

where N is the size of $\mathcal{I}$ (number of constants in $\mathcal{I}$), and m the size of the query (number of distinct variables in Q).

Boolean Conjunctive Query

$$true \leftarrow R_1(\vec{U_1}), \ldots, R_n(\vec{U_n}).$$

- Boolean conjunctive query answering is a decision problem.
- The complexity of the boolean conjunctive query answering is NP-complete.

Decision Problems

- Problems where the answer is "yes" or "no"
- Formally,
  - A language L over some alphabet Σ.
  - An *instance* is given as a word $x \in \Sigma^*$.
  - Question: whether $x \in L$ holds
- The resources (i.e., either time or space) required in the worst case to find the correct answer for any instance x of a problem L is referred to as the *complexity* of the problem L

Complexity classes

$$L \subseteq NL \subseteq P \subseteq NP \subseteq PSPACE \subseteq EXPTIME \subseteq NEXPTIME$$

These are the classes of problems which can be solved in

- logarithmic space (L),
- non-deterministic logarithmic space (NL),
- polynomial time (P),
- non-deterministic polynomial time (NP),
- polynomial space (PSPACE),
- exponential time (EXPTIME), and
- non-deterministic exponential time (NEXPTIME).

Complexity classes – co Problems

- Any complexity class $\mathcal{C}$ has its *complementary class* denoted by co-$\mathcal{C}$
- For every language $L \subseteq \Sigma^*$, let $\overline{L}$ denote its *complement*, i.e. the set $\Sigma^* \setminus L$. Then co-$\mathcal{C}$ is $\{\overline{L} \mid L \in \mathcal{C}\}$.
- Every deterministic complexity class is closed under complement, because one can simply add a last step to the algorithm which reverses the answer. (co-P?)

Complexity classes – Reductions

- **Logspace Reduction**
  - Let L_1 and L_2 be decision problems (languages over some alphabet Σ).
  - $R : \Sigma^* \to \Sigma^*$ be a function which can be computed in logarithmic space
  - The following property holds: for every $x \in \Sigma^*$, $x \in L_1$ iff $R(x) \in L_2$.
  - Then R is called a *logarithmic-space reduction* from L_1 to L_2 and we say that L_1 is *reducible* to L_2.
- **Hardness, Completeness**
  Let $\mathcal{C}$ be a set of languages. A language L is called *$\mathcal{C}$-hard* if any language L' in $\mathcal{C}$ is reducible to L. If L is $\mathcal{C}$-hard and $L \in \mathcal{C}$ then L is called *complete for $\mathcal{C}$* or simply *$\mathcal{C}$-complete*.

Turing machines

A *deterministic Turing machine (DTM)* is defined as a quadruple

$$(S, \Sigma, \delta, s_0)$$

- S is a finite set of *states*,
- Σ is a finite alphabet of *symbols*, which contains a special symbol $\sqcup$ called the *blank*.
- δ is a *transition function*,
- and $s_0 \in S$ is the *initial state*.

The transition function δ is a map

$$\delta : \quad S \times \Sigma \quad \rightarrow \quad (S \cup \{\texttt{yes}, \texttt{no}\}) \times \Sigma \times \{\texttt{-1}, \ \texttt{0}, \ \texttt{+1}\},$$

where `yes`, and `no` denote two additional states not occurring in S, and `-1`, `0`, `+1` denote *motion directions*.

Turing machines

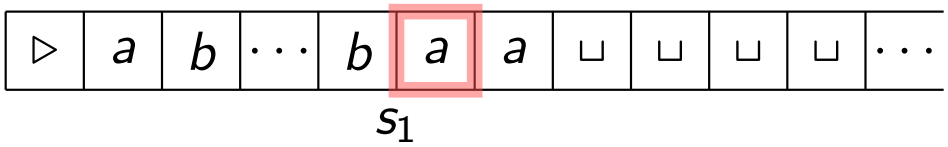
DTM quadruple:

$$(\Sigma, S, \delta, s_0)$$

Transition function:

$$\delta(s, \sigma) = (s', \sigma', d).$$

The tape of the TM



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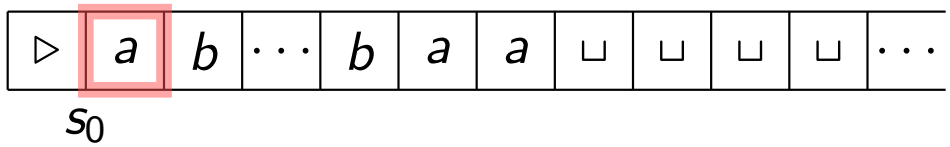

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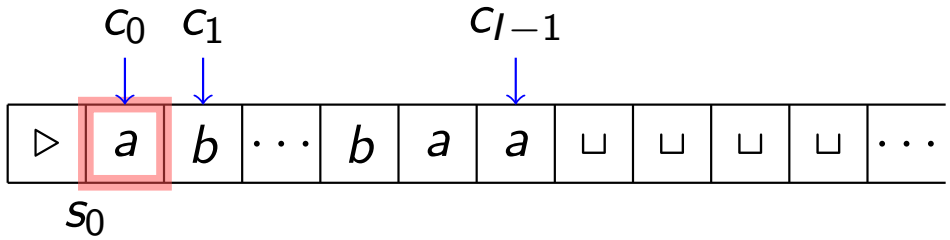

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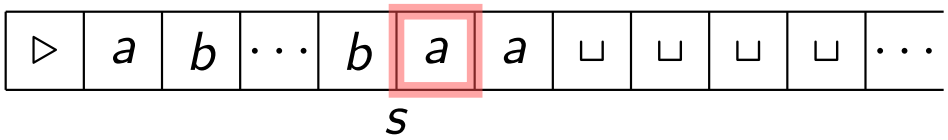

Transition Function example:

$$\delta(s, a) = (s', b, -1)$$

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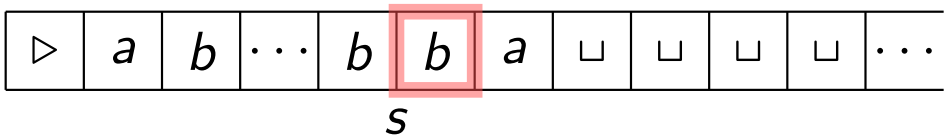

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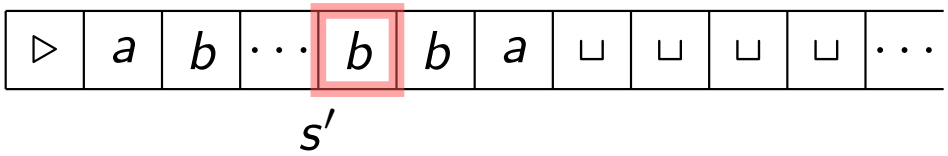

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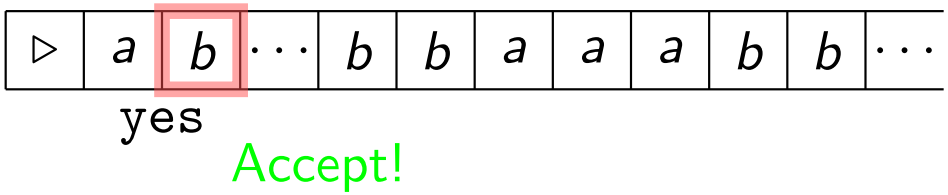

T halts, when any of the states `yes` or `no` is reached

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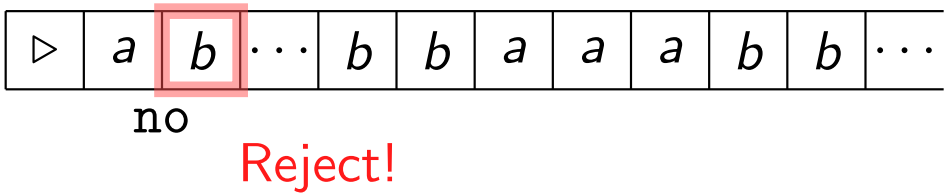


T halts, when any of the states `yes` or `no` is reached

NDTM

A *non-deterministic Turing machine (NDTM)* is defined as a quadruple

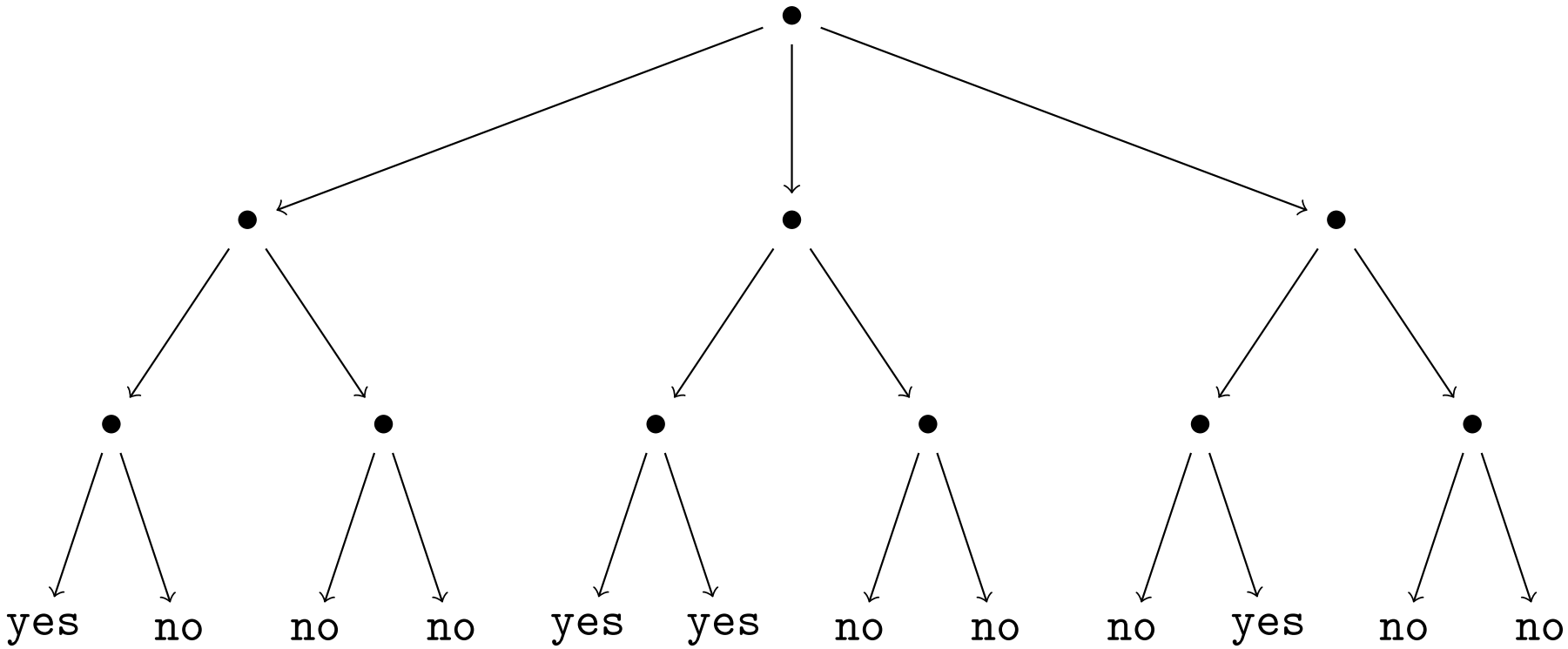
$$(S, \Sigma, \Delta, s_0)$$

- S, Σ, s_0 are the same as DTM
- Δ is no longer a function, but a relation:

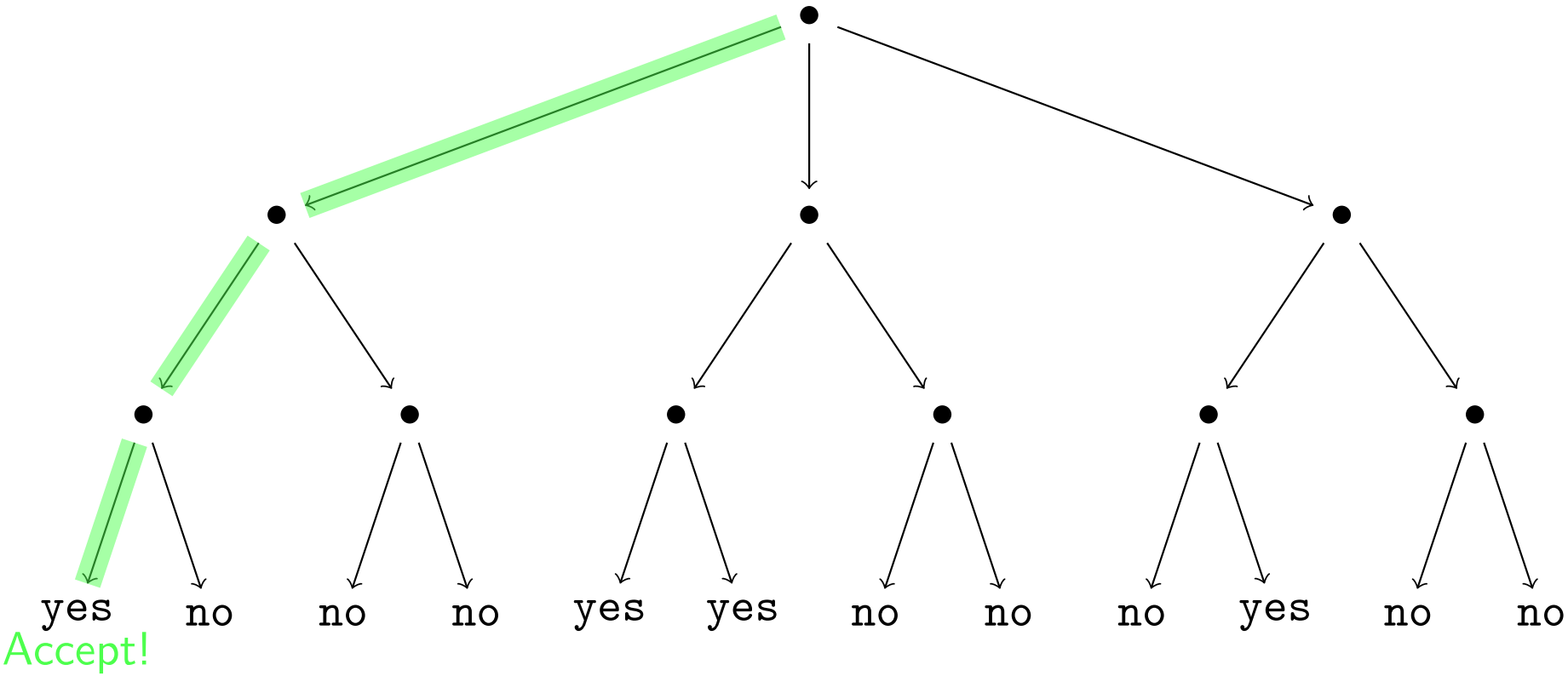
$$\Delta \subseteq (S \times \Sigma) \times (S \cup \{\texttt{yes}, \texttt{no}\}) \times \Sigma \times \{\texttt{-1}, \texttt{ 0}, \texttt{ +1}\}.$$

- A tuple with s and σ. If the number of such tuples is greater than one, the NDTM non-deterministically chooses any of them and operates accordingly.
- Unlike the case of a DTM, the definition of acceptance and rejection by a NDTM is asymmetric.

Nondeterministic Computation (Accept)



Nondeterministic Computation (Accept)



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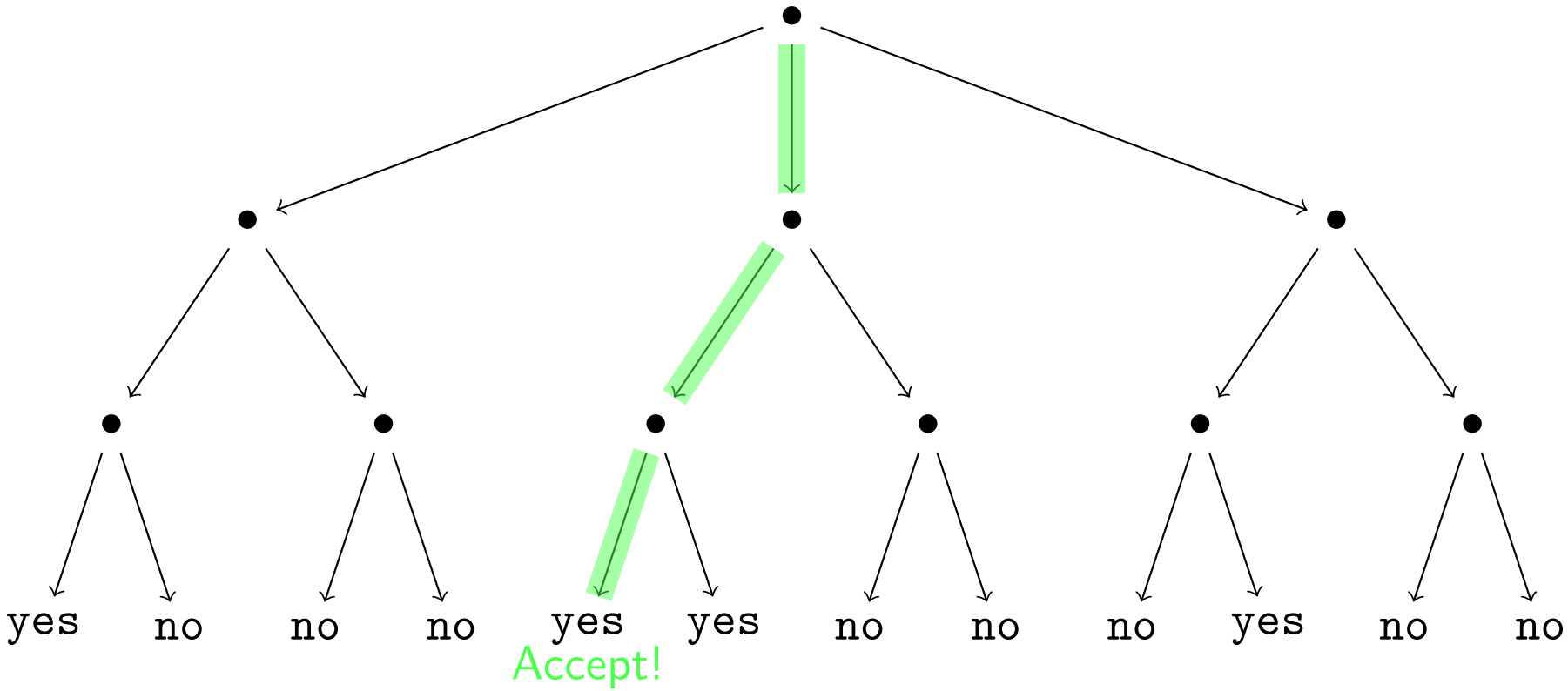
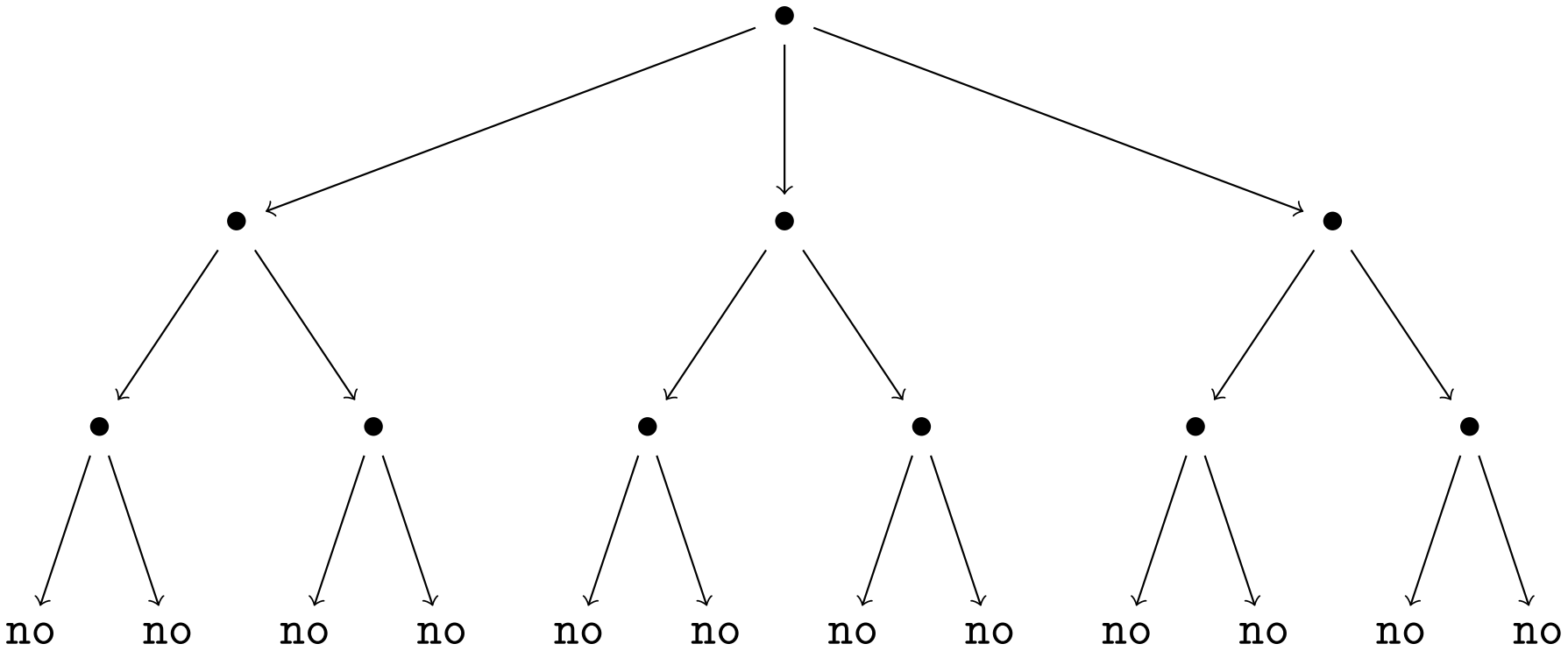
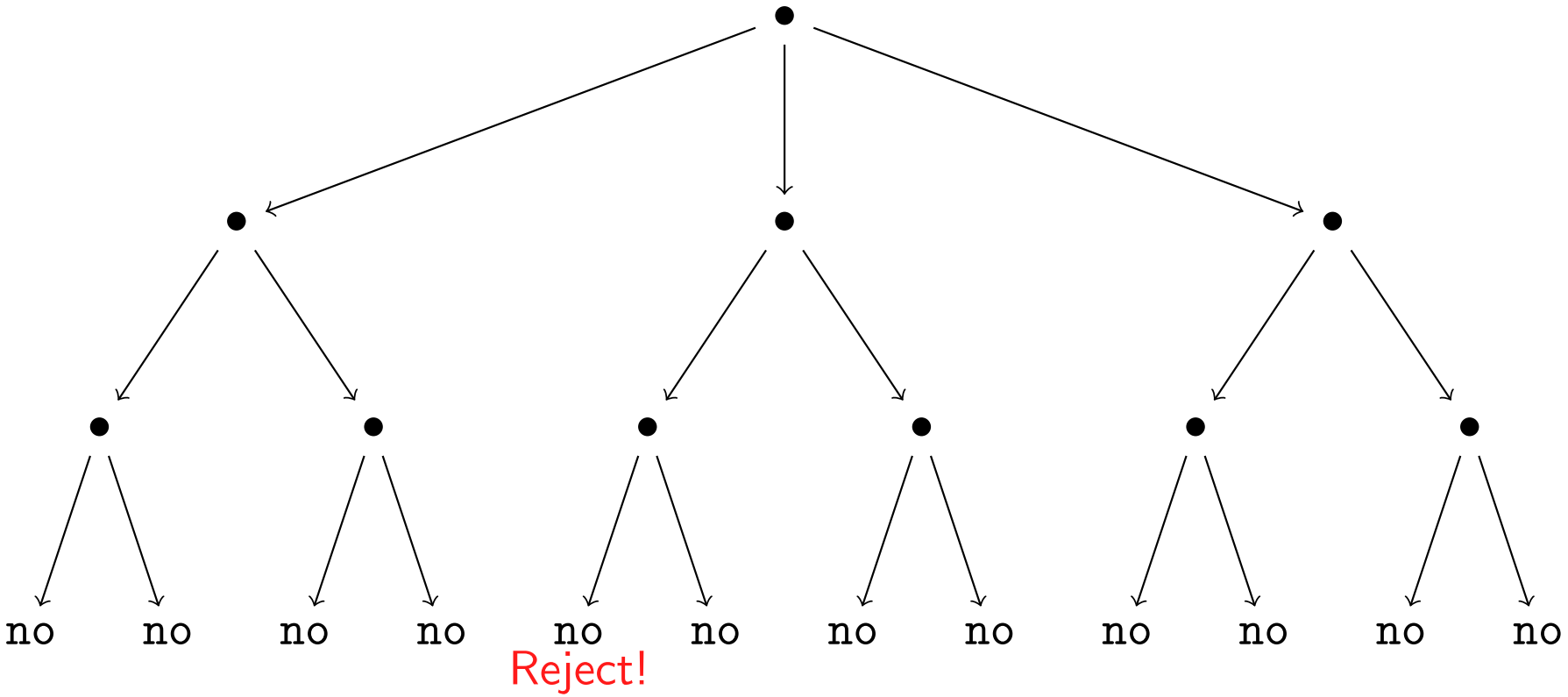


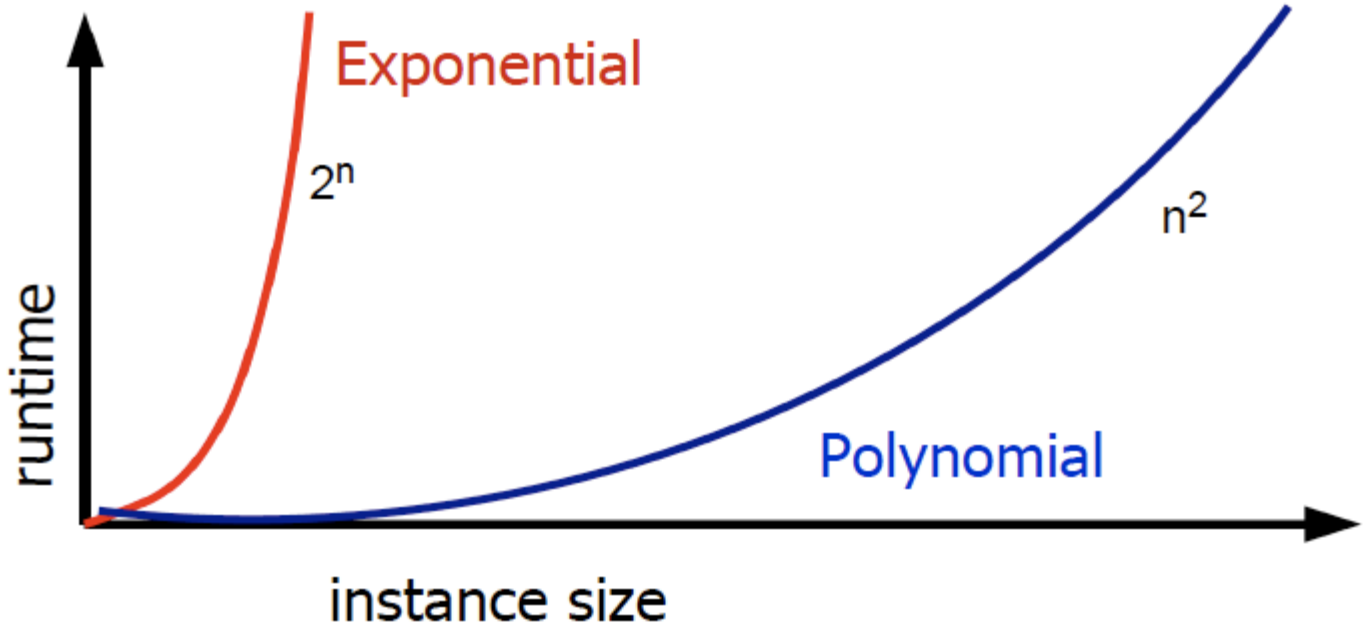
Nondeterministic Computation (Rejection)



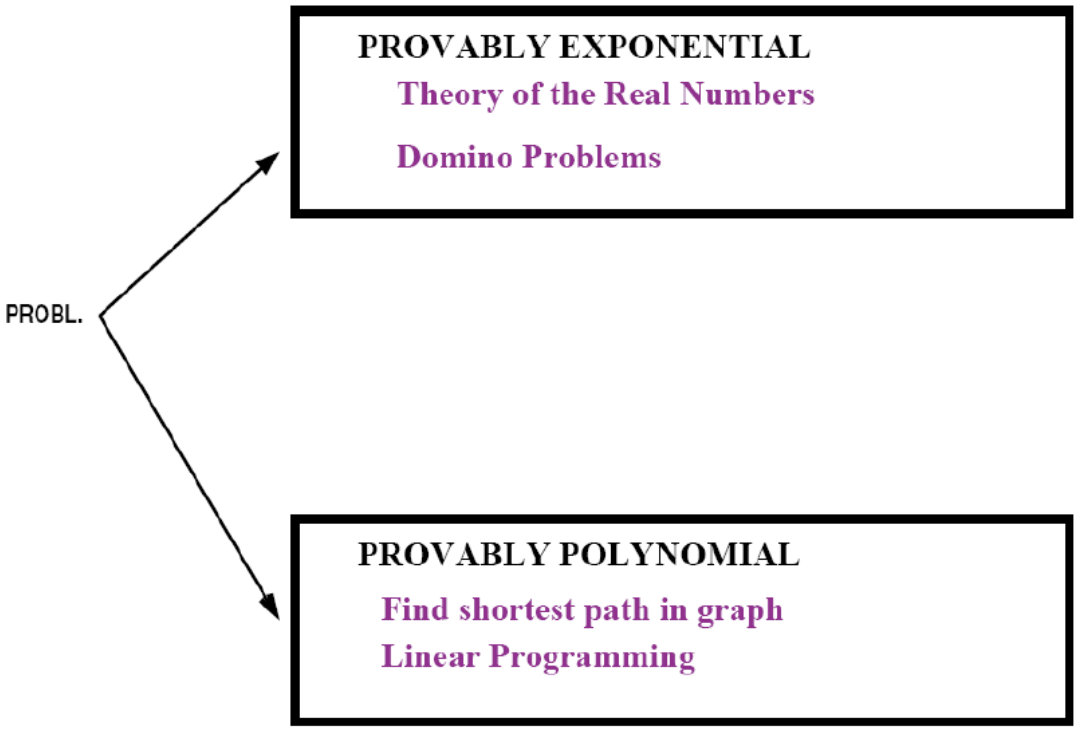
Nondeterministic Computation (Rejection)



Exponentail vs. Polynomial



NP problems


PROVABLY EXPONENTIAL
Theory of the Real Numbers
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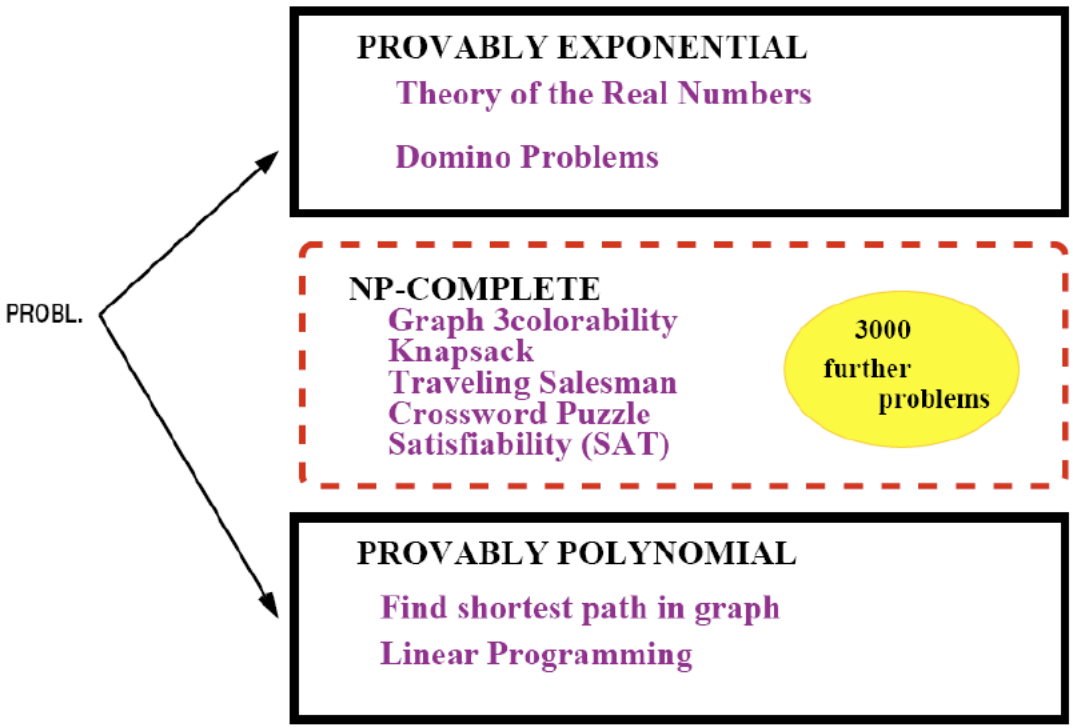
NP problems


PROVABLY EXPONENTIAL
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NP-COMPLETE
Graph 3colorability
Knapsack
Traveling Salesman
Crossword Puzzle
Satisfiability (SAT)
3000
further
problems
PROVABLY POLYNOMIAL
Find shortest path in graph
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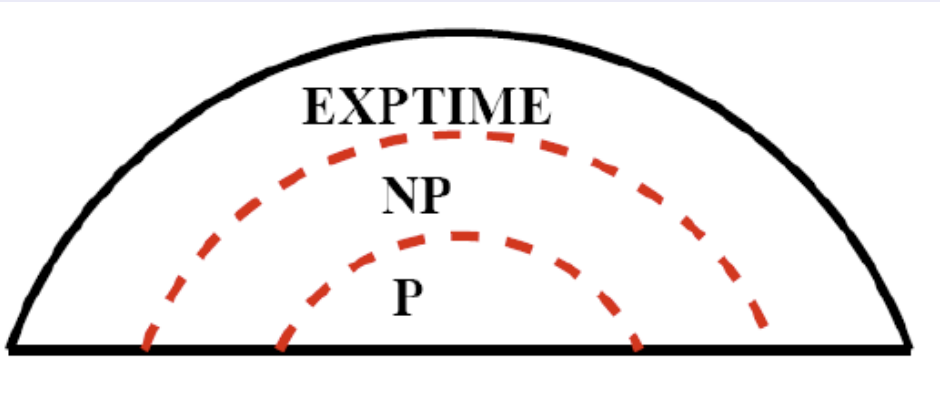
NP problems

Paradigm: Guess and Check

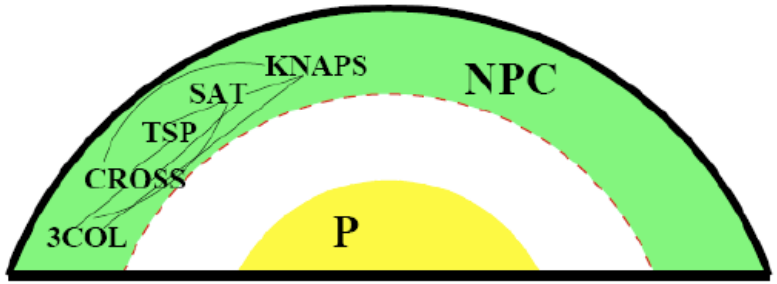
There are many problems in NP for which the best known algorithm runs in exponential time only.

However, for no problem in NP there is a proof of exponential complexity.



NP Complete Problems

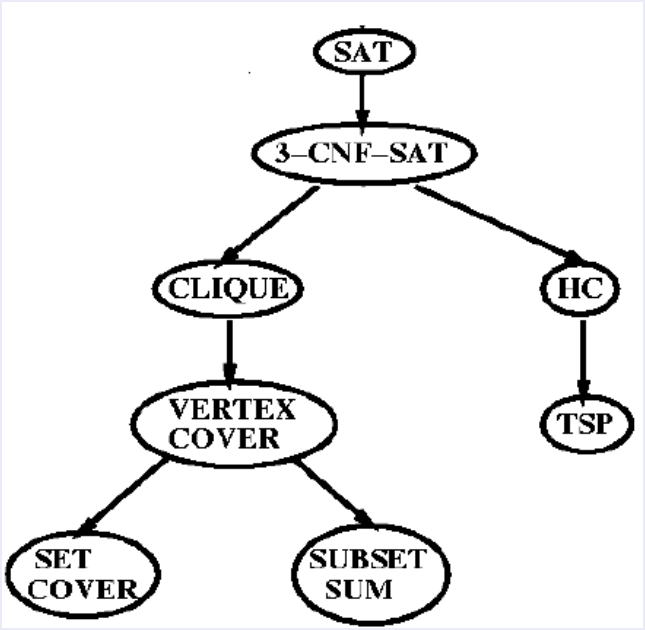
NP-Complete: the hardest problems in NP
All problems in NPC can be polynomially transformed into one another.
One polynomially solvable $\rightarrow$ all polynomially solvable, i.e. NP=P.



Structure inside NP

Karp and Cook's Theorem [1972]

SAT problem is NP-Complete.



$\rightarrow$ Thousands of other problems.

P = NP?

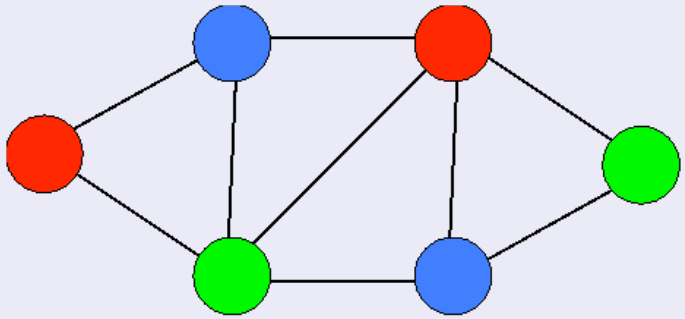
The most important open problem of Theoretical Computer Science.
Clay Mathematical Institute one million dollars for solving it.

NP Complete Problems

- 3-SAT

$$(v_1 \vee v_2 \vee \bar{v}_3) \wedge (v_1 \vee \bar{v}_2 \vee \bar{v}_4) \wedge (\bar{v}_1 \vee v_3 \vee v_4)$$

- 3-Colorability



- TSP (Travelling Salesman Problem)

NP-completeness proof

- The problem is in NP:
  - Guess a substitution (mapping) from all the variables in Q to a set of constants in $\mathcal{I}$,
  - Check whether the substitution makes the subgoals in the body true.
- The problem is NP hard: reduction from 3-SAT.

$$(v_1 \vee v_2 \vee \bar{v_3}) \wedge (v_1 \vee \bar{v_2} \vee \bar{v_4}) \wedge (\bar{v_1} \vee v_3 \vee v_4)$$

NP-completeness proof

The problem is NP hard: reduction from 3-SAT.

$$(v_1 \vee v_2 \vee \bar{v}_3) \wedge (v_1 \vee \bar{v}_2 \vee \bar{v}_4) \wedge (\bar{v}_1 \vee v_3 \vee v_4)$$

$\mathcal{I}$: $r(1,1,1)$, $r(1,1,0)$, $r(1,0,1)$, $r(1,0,0)$, $r(0,1,1)$, $r(0,1,0)$, $r(0,0,1)$, $c(1,0)$, $c(0,1)$.
Q: $r(v_1, v_2, nv_3)$, $r(v_1, nv_2, nv_4)$, $r(nv_1, v_3, v_4)$, $c(v_1, nv_1)$, $c(v_2, nv_2)$, $c(v_3, nv_3)$, $c(v_4, nv_4)$.

Equivalence proof $\rightarrow$ Homework.

Problemes

Let Q, Q_1, Q_2 be conjunctive queries.

Containment: $Q_1 \sqsubseteq Q_2$, i.e., $Q_1(\mathcal{I}) \subseteq Q_2(\mathcal{I})$ for any instance $\mathcal{I}$?

Equivalence: $Q_1 \equiv Q_2$, i.e., $Q_1 \sqsubseteq Q_2$ and $Q_2 \sqsubseteq Q_1$?

Minimization: Given Q_1, construct an equivalent query Q_2, which has as most as much subgoals in its body as Q_1 and is minimal in the sense, that any query Q_3 being equivalent to Q_2 has at least as much subgoals in the body as Q_2.

Q_2 is called *minimal*.

Example

$$Sales(Part, Supplier, Customer),$$
$$Part(PName, Type),$$
$$Cust(CName, CAddr),$$
$$Supp(SName, SAddr).$$

Equivalent queries:

$$Q: \quad ans(T) \leftarrow Sales(P, S, C), Part(P, T), Cust(C, A), Supp(S, A)$$

$$Q': \quad ans(T) \leftarrow Sales(P, S, C), Part(P, T), Cust(C, A), Supp(S, A), Sales(P', S', C'), Part(P', T)$$

Lemma

Let

$$Q_1 : \quad ans(\vec{U}) \leftarrow R_1(\vec{U_1}), \ldots, R_n(\vec{U_n})$$
$$Q_2 : \quad ans(\vec{U}) \leftarrow S_1(\vec{V_1}), \ldots, S_m(\vec{V_m})$$

be conjunctive queries, where

$$\{R_1(\vec{U_1}), \ldots, R_n(\vec{U_n})\} \supseteq \{S_1(\vec{V_1}), \ldots, S_m(\vec{V_m})\}$$

Then $Q_1 \sqsubseteq Q_2$.

Substitution

- A *substitution* θ over a set of variables $\mathcal{V}$ is a mapping from $\mathcal{V}$ to $\mathcal{V} \cup \mathbf{dom}$, where **dom** a corresponding domain.
- We extend θ to constants $a \in \mathbf{dom}$ and relation names $R \in \mathcal{R}$, where $\theta(a) = a$, resp. $\theta(R) = R$.

Example

Consider

$$Q: \quad ans(T) \leftarrow Sales(P, S, C), Part(P, T), Cust(C, A), Supp(S, A)$$

$$Q': \quad ans(T) \leftarrow Sales(P, S, C), Part(P, T), Cust(C, A), Supp(S, A), Sales(P', S', C'), Part(P', T)$$

and θ:

X	P	P'	S	S'	C	C'	T	A
$\theta(X)$	P	P	S	S	C	C	T	A

Containment Mapping

Given conjunctive queries

$$Q_1 : \quad ans(\vec{U}) \leftarrow R_1(\vec{U_1}), \ldots, R_n(\vec{U_n})$$
$$Q_2 : \quad ans(\vec{V}) \leftarrow S_1(\vec{V_1}), \ldots, S_m(\vec{V_m})$$

Substitution θ is called *containment mapping* from Q_2 to Q_1, if Q_2 can be transformed by means of θ to become Q_1:

- $\theta(ans(\vec{V})) = ans(\vec{U})$,
- for $i = 1, \ldots, m$ there exists a $j \in \{1, \ldots, n\}$, such that $\theta(S_i(\vec{V_i})) = R_j(\vec{U_j})$.

Example

$$Q: \quad ans(T) \leftarrow Sales(P, S, C), Part(P, T), Cust(C, A), Supp(S, A)$$

$$Q': \quad ans(T) \leftarrow Sales(P, S, C), Part(P, T), Cust(C, A), Supp(S, A), Sales(P', S', C'), Part(P', T)$$

θ:

X	P	P'	S	S'	C	C'	T	A
$\theta(X)$	P	P	S	S	C	C	T	A

θ is a containment mapping.

Theorem

Let

$$Q_1 : \quad ans(\vec{U}) \leftarrow R_1(\vec{U_1}), \ldots, R_n(\vec{U_n})$$
$$Q_2 : \quad ans(\vec{V}) \leftarrow S_1(\vec{V_1}), \ldots, S_m(\vec{V_m})$$

be conjunctive queries.
$Q_1 \sqsubseteq Q_2$ iff there exists a containment mapping θ from Q_2 to Q_1.

Proof "$\Leftarrow$":

There exists containment mapping θ.
Let $\mathcal{I}$ be an instance of Q_1 and let $\mu \in Q_1(\mathcal{I})$.
There exists a substitution τ, such that $\tau(\vec{U}_j) \in \mathcal{I}(R_j)$, $j \in \{1, \ldots, n\}$ and $\mu = \tau(\vec{U})$.
Consider a substitution $\tau' = \tau \circ \theta$ and further $\tau'(S_i(\vec{V}_i))$.
There holds $\tau'(\vec{V}_i) \in \mathcal{I}(S_i)$, $i \in \{1, \ldots, m\}$ and therefore also $\mu = \tau'(\vec{V})$.
D.h., $\mu \in Q_2(\mathcal{I})$.

Canonical Instance

Let Q be a conjunctive $ans(\vec{U}) \leftarrow R_1(\vec{U_1}), \ldots, R_n(\vec{U_n})$ over a database schema $\mathcal{R}$. The *canonical instance* $\mathcal{I}_Q$ to Q is constructed as follows.

$\mathcal{I}_Q$ is an instance of $\mathcal{R} = \{R_1, \ldots, R_n\}$.

Let τ be a substitution, which assigns to any X in Q an unique constant a_X.

- For any literal $R(t_1, \ldots, t_n)$ in the body, insert a tupel of the form $(\tau(t_1), \ldots, \tau(t_n))$ into $\mathcal{I}_Q(R)$; we also write $\tau(R(t_1, \ldots, t_n)) \in \mathcal{I}_Q(R)$.

  No other tuples are inserted into $\mathcal{I}_Q(R)$.

τ is called *canonical substitution*.

Example

$$Q: \quad ans(T) \leftarrow Sales(P, S, C), Part(P, T), Cust(C, A), Supp(S, A)$$

$$Q': \quad ans(T) \leftarrow Sales(P, S, C), Part(P, T), Cust(C, A), Supp(S, A), Sales(P', S', C'), Part(P', T)$$

$\mathcal{I}_Q$:

Sales			Part		Cust		Supp	
a_P	a_S	a_C	a_P	a_T	a_C	a_A	a_S	a_A

$\mathcal{I}_{Q'}$:

Sales			Part		Cust		Supp	
a_P	a_S	a_C	a_P	a_T	a_C	a_A	a_S	a_A
$a_{P'}$	$a_{S'}$	$a_{C'}$	$a_{P'}$	a_T				

Proof "$\Rightarrow$":

$Q_1 \sqsubseteq Q_2$.
Consider $\mathcal{I}_{Q_1}$ and the corresponding canonical substitution τ.
Then $\tau(ans(\vec{U})) \in Q_1(\mathcal{I}_{Q_1})$.
Because of $Q_1 \sqsubseteq Q_2$ further $\tau(ans(\vec{U})) \in Q_2(\mathcal{I}_{Q_1})$.
Thus, there exists a substitution ρ, such that $\rho(S_i(\vec{V}_i)) = \tau(R_j(\vec{U}_j))$, $1 \leq i \leq m$, $j \in \{1, \ldots, n\}$ und $\rho(ans(\vec{V})) = \tau(ans(\vec{U}))$.
$\tau^{-1} \circ \rho$ is a containment mapping.

Corollary

Let

$$Q_1 : \quad ans(\vec{U}) \leftarrow R_1(\vec{U_1}), \ldots, R_n(\vec{U_n})$$
$$Q_2 : \quad ans(\vec{V}) \leftarrow S_1(\vec{V_1}), \ldots, S_m(\vec{V_m})$$

be conjunctive queries, $\mathcal{I}_{Q_1}$ the canonical instance to Q_1 with canonical substitution τ.
$Q_1 \sqsubseteq Q_2$, iff $\tau(ans(\vec{U})) \in Q_2(\mathcal{I}_{Q_1})$.

Proof: We show, whenever $\tau(ans(\vec{U})) \in Q_2(\mathcal{I}_{Q_1})$, then $Q_1 \sqsubseteq Q_2$.
For any S_j in Q_2, $\mathcal{I}_{Q_1}$ is not empty. Therefore, for S_j there exists a R_i, such that $S_j = R_i$. Further, there exists a substitution ρ, such that for $S_j(\vec{V_j})$ we have $\rho(V_j) \in \mathcal{I}_{Q_1}(R_i)$. $\rho \circ \tau^{-1}$ is a containment mapping from Q_2 to Q_1.

Example

$$ans(a_T) \in Q(\mathcal{I}_{Q'})$$

and

$$ans(a_T) \in Q'(\mathcal{I}_Q).$$